Quiz 5: Mach-Zender Adjacency Matrix
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Lasers and Optomechanics

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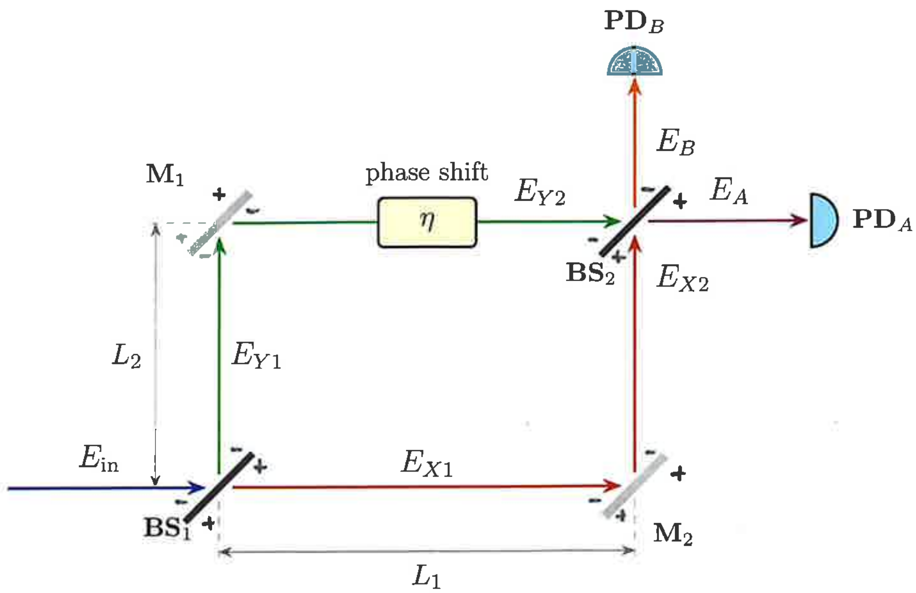
Mach-Zender Interferometer

Suppose we have a Mach-Zender interferometer as labeled below, sourced with a input field E_in with wavenumber k and with side lengths L_1 and L_2. In the Y-arm of the Mach-Zender is an optical element that applies a phase-shift η to the beam that passes through. Assume plane-waves for all fields in the interferometer.



For the below questions, use the following field vector $\vec{E}$ to represent the different seven labeled fields of the interferometer:

$$\vec{E} = \begin{bmatrix} E_{\text{in}} & E_{X1} & E_{Y1} & E_{X2} & E_{Y2} & E_A & E_B \end{bmatrix}^T \tag{1}$$

You may also want to use

$$\cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}, \qquad \sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i} \tag{2}$$

1. Write six equations relating the seven fields of the Mach-Zender interferometer.
   Use parameters $L_1, L_2, \eta, r_{BS1}, t_{BS1}, r_{BS2}, t_{BS2}, r_{M1}, r_{M2}$.

2. Find expressions for field transfer functions $\dfrac{E_A}{E_{\text{in}}}$ and $\dfrac{E_B}{E_{\text{in}}}$.
   Assume that $r_{BS1} = t_{BS1} = r_{BS2} = t_{BS2} = \dfrac{1}{\sqrt{2}}$, and that $r_{M1} = r_{M2} = 1$.

3. Using your result from (2), calculate the power relationships $\dfrac{P_A}{P_{\text{in}}}$ and $\dfrac{P_B}{P_{\text{in}}}$.

4. What would a change in L_1 or L_2 do to E_A and E_B? What about the power expressions P_A and P_B?

5. What about a change in η? Would this impact E_A and E_B or P_A and P_B?

6. Using your equations from (1), put together an adjacency matrix for the Mach-Zender interferometer.

1. $E_{x1} = t_{BS1}\, e^{-ikL_1}\, E_{in}$

$$E_{y1} = -r_{BS1}\, e^{-ikL_2}\, E_{in}$$

$$E_{x2} = -r_{M2}\, e^{-ikL_2}\, E_{x1}$$

$$E_{y2} = -r_{M1}\, e^{-ikL_1}\, e^{i\eta}\, E_{y1}$$

$$E_A = r_{BS2}\, E_{x2} + t_{BS2}\, E_{y2}$$

$$E_B = t_{BS2}\, E_{x2} - r_{BS2}\, E_{y2}$$

2. $E_A = -r_{BS2}\, r_{M2}\, t_{BS1}\, e^{-ik(L_1+L_2)}\, E_{in} + t_{BS2}\, r_{M1}\, r_{BS1}\, e^{-ik(L_1+L_2)}\, e^{i\eta}\, E_{in}$

$$E_B = -\underbrace{t_{BS2}}_{1/\sqrt{2}}\, \underbrace{r_{M2}}_{1}\, \underbrace{t_{BS1}}_{1/\sqrt{2}}\, e^{-ik(L_1+L_2)}\, E_{in} - \underbrace{r_{BS2}}_{1/\sqrt{2}}\, \underbrace{r_{M1}}_{1}\, \underbrace{r_{BS1}}_{1/\sqrt{2}}\, e^{-ik(L_1+L_2)}\, e^{i\eta}\, E_{in}$$

$$\frac{E_A}{E_{in}} = \frac{1}{2}\, e^{-ik(L_1+L_2)} \left[-1 + e^{i\eta}\right] = \frac{1}{2}\, e^{-ik(L_1+L_2)}\, e^{i\eta/2} \left[-e^{-i\eta/2} + e^{i\eta/2}\right]$$

$$\frac{E_B}{E_{in}} = \frac{1}{2}\, e^{-ik(L_1+L_2)} \left[-1 - e^{i\eta}\right] = -\frac{1}{2}\, e^{-ik(L_1+L_2)}\, e^{i\eta/2} \left[e^{-i\eta/2} + e^{i\eta/2}\right]$$

$$\frac{E_A}{E_{in}} = i\, e^{-ik(L_1+L_2)}\, e^{i\eta/2}\, \sin(\eta/2)$$

$$\frac{E_B}{E_{in}} = -\, e^{ik(L_1+L_2)}\, e^{i\eta/2}\, \cos(\eta/2)$$

3. $\dfrac{P_A}{P_{in}} = \sin^2(\eta/2)$

$$\frac{P_B}{P_{in}} = \cos^2(\eta/2)$$

4. Change in L_1 or L_2 is common motion, so yields only a phase rotation in E_A or E_B. No change in P_A or P_B would occur.

5. Change in η would be both common + differential. This rotates + scales $E_A + E_B$

5. (continued). P_A and P_B would see a change in magnitude like $\sin^2(\eta/2)$ and $\cos^2(\eta/2)$.

6. Adjacency Matrix

$$
\begin{bmatrix} E_{in} \\ E_{x1} \\ E_{y1} \\ E_{x2} \\ E_{y2} \\ E_A \\ E_B \end{bmatrix}
=
\begin{array}{c}
\begin{array}{ccccccc} \quad E_{in} \quad & \quad E_{x1} \quad & \quad E_{y1} \quad & E_{x2} & E_{y2} & E_A & E_B \end{array} \\
\left[\begin{array}{c:c:c|c:c|c:c}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
t_{BS1} e^{-ikL_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
-r_{BS1} e^{-ikL_2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -r_{M2} e^{-ikL_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -r_{M1} e^{-ikL_1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & r_{BS2} & t_{BS2} & 0 & 0 \\
0 & 0 & 0 & t_{BS2} & -r_{BS2} & 0 & 0
\end{array}\right]
\end{array}
$$