Quiz 4: Scattering Matrices and Fabry-Perots
February 18, 2026
Lasers and Optomechanics

Name: ____

General Scattering Matrix

In class we discussed a 2×2 general scattering matrix $\boldsymbol{S}$. If we assume the scattering matrix is

1. unitary, and
2. time-reversable,

we can write the general scattering matrix as

$$\begin{aligned} \boldsymbol{E}_{\text{out}} &= \qquad\qquad \boldsymbol{S} \qquad\qquad \boldsymbol{E}_{\text{in}} \\ \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}_{\text{out}} &= e^{i\phi} \begin{bmatrix} \sqrt{1-r^2} & re^{i\delta} \\ -re^{-i\delta} & \sqrt{1-r^2} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}_{\text{in}} \end{aligned} \tag{1}$$

where $r \in [0, 1]$ and $\phi, \delta \in [0, 2\pi]$.

Write the parameters r, ϕ, δ needed to express the following matrices:

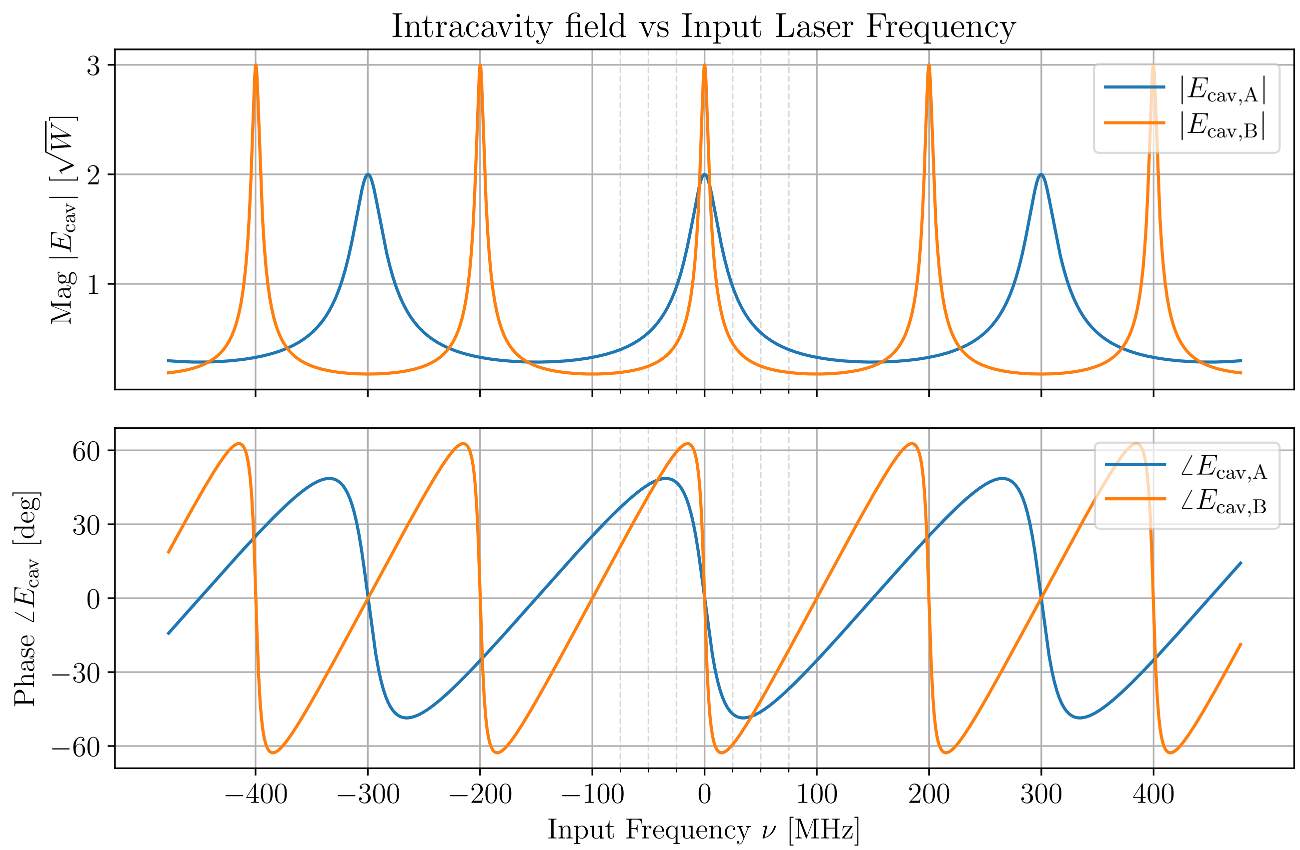
1. $\begin{bmatrix} t & r \\ -r & t \end{bmatrix}$

2. $\begin{bmatrix} e^{-ikL} & 0 \\ 0 & e^{-ikL} \end{bmatrix}$

3. $\begin{bmatrix} it & r \\ r & it \end{bmatrix}$

Laser Amplifier Scattering Matrix

1. Write a scattering matrix for a single-pass laser amplifier with power gain G and length L.
2. Which assumption about scattering matrices made above would no longer be valid?



Two Cavities Swept by One Laser

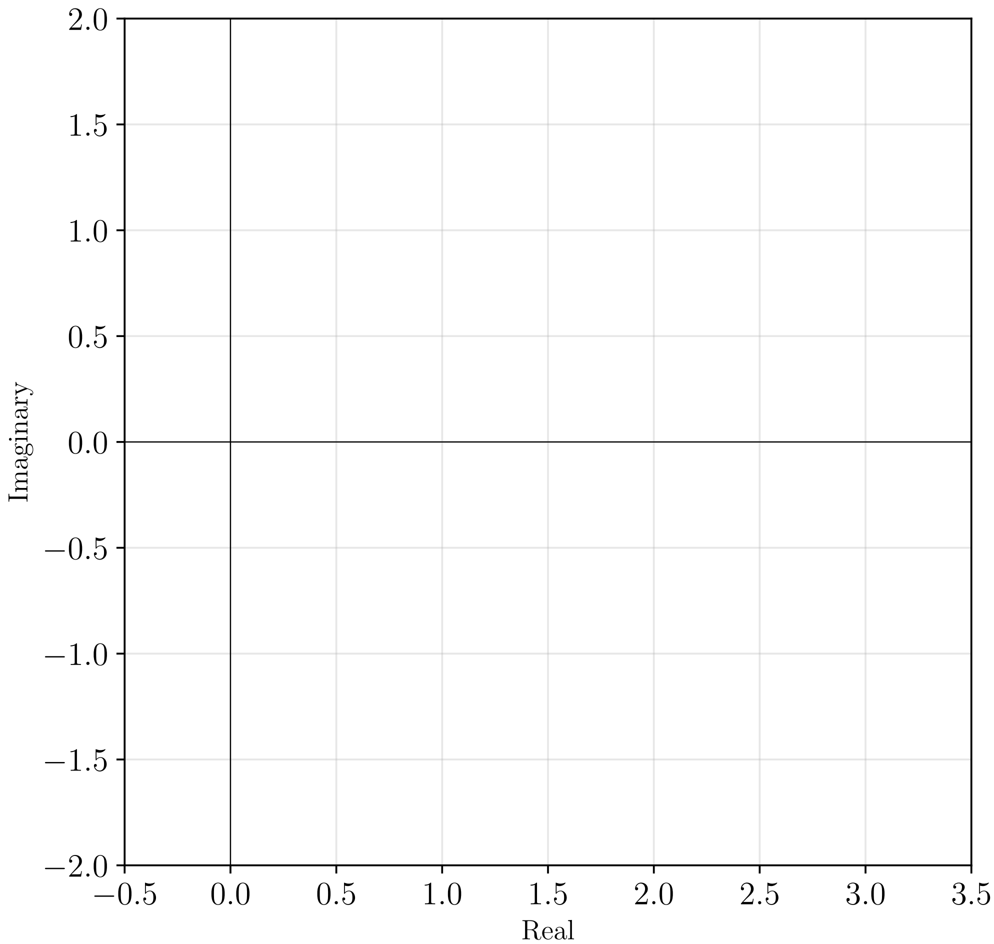
In the above figure, two cavities A and B are swept by the same laser with frequency offset ν. The intracavity fields are plotted during this sweep. For the below questions, recall the following equations,

$$\frac{E_{\rm cav}}{E_{\rm in}} = \frac{t_1}{1 - r_1 r_2 e^{-i\phi}} \tag{2}$$

where ϕ is the round-trip phase.

1. What is the *free spectral range* FSR for the two cavities?
2. Which cavity has the longer length, A or B?
3. What is the *finesse* $\mathcal{F}$ for cavity A?
4. Assuming the cavities are *critically-coupled*, what is the power transmission $T = t^2$ of the mirrors?
5. Calculate the *discriminant* of the intracavity field with respect to the round-trip phase ϕ: $\dfrac{dE_{\rm cav}}{d\phi}$.
6. Draw the phasor for cavity B on the plot on the next page.

   Draw the vectors for $E_{\rm cav}$ and its discriminant $\dfrac{dE_{\rm cav}}{d\phi}$ evaluated at resonance $\phi = 0$.
7. Is the discriminant purely a magnitude change, a phase change, or a mixture of both at resonance? What about in general?



2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Imaginary
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Real