Quiz 3: Lasers

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Lasers and Optomechanics

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Components of a Laser

What are the three components of any laser?

1. Gain Medium
2. Pump
3. Laser Oscillator

Stimulated Emission Copies

What are the four aspects of incoming laser light copied in stimulated emission?

1. Phase
2. Frequency
3. Momentum
4. Polarization

Absorption

We can model the absorption $\alpha(\omega)$ of an electric field $E(z) = E_0 e^{-\alpha(\omega) z}$ over distance z through some medium, where

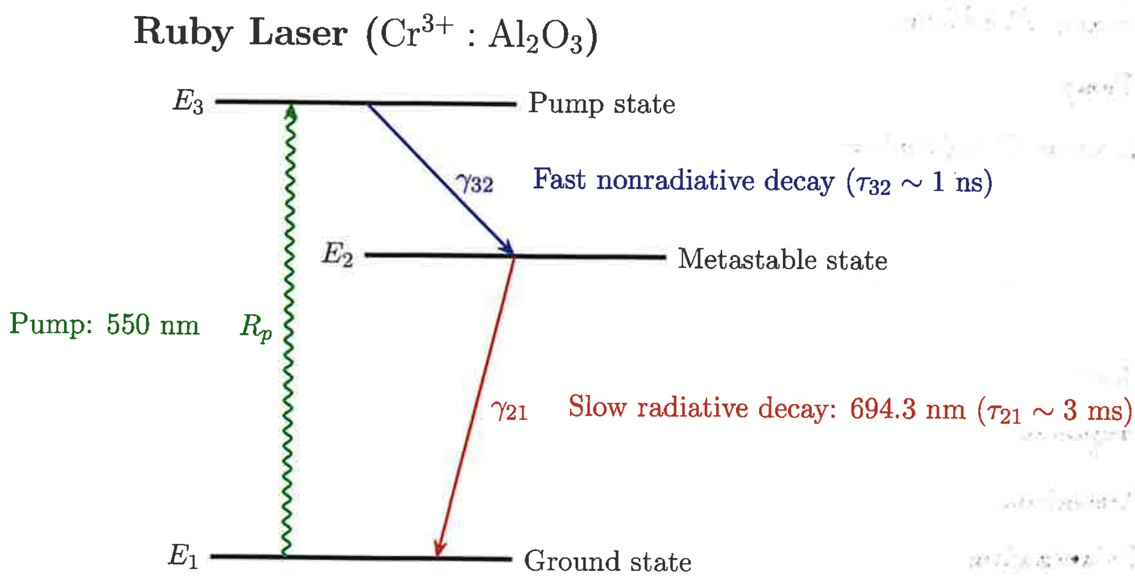
$$\alpha(\omega) = \frac{\lambda^2}{4\pi} \frac{\gamma_{\text{rad}}}{\Delta\omega_a} \frac{N_1 - N_2}{1 + 4\left(\frac{\omega_{21} - \omega}{\Delta\omega_a}\right)^2} \tag{1}$$

where λ is the laser wavelength,
γ_{rad} is the radiative decay rate,
$\Delta\omega_a$ is the atomic linewidth,
and ω_{21} is the center frequency of the atomic line.

1. What happens to our absorption expression in Eq. 1 if $N_2 > N_1$? Becomes negative, can emit
2. What is this status when $N_2 > N_1$? Population Inversion

Three Level Atomic Rate Equations for Pumped System

The three-level lasing system for ruby is reproduced below.
Suppose the occupancies for the three energy levels E_1, E_2, E_3 are N_1, N_2, N_3.
Assume we have some pump transition rate R_p from the first energy level to the third energy level, as well as relaxation rates γ_{32} and γ_{21} for transitions $E_3 \to E_2$ and $E_2 \to E_1$.
Assume no lasing action.



1. What is the *quantum defect* associated with ruby?

2. Write out the atomic rate equation for the third energy level occupancy $\dfrac{dN_3}{dt}$, assuming a pump but no lasing. Only include the $3 \to 2$ spontaneous emission rate γ_{32}
(i.e. don't worry about the spontaneous emission to the ground state γ_{31}).

3. Write out the atomic rate equation for the second energy level occupancy $\dfrac{dN_2}{dt}$, assuming no lasing.

4. Write out the atomic rate equation for the second energy level occupancy $\dfrac{dN_1}{dt}$, assuming no lasing.

5. Assume now your pumped system has reached steady-state.
What are the steady-state occupancy ratios $\dfrac{N_1}{N_3}$ and $\dfrac{N_2}{N_3}$ of your system?

6. Calculate $\dfrac{N_2 - N_1}{N_3}$ for the steady state.

7. When is population inversion possible for your result above?

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$c = \lambda \nu$

$\omega = 2\pi\nu = \frac{2\pi c}{\lambda}$

$E = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{hc}{E}$

1. $E_3 - E_2 = \hbar(\omega_3 - \omega_2) = 2\pi\hbar c\left(\frac{1}{\lambda_3} - \frac{1}{\lambda_2}\right)$

$$= hc\left(\frac{1}{\lambda_3} - \frac{1}{\lambda_2}\right)$$

$\lambda_2 = 694.3\ \text{nm}$

$$\lambda_3 = \frac{hc}{E_3 - E_1} - \frac{hc}{E_2 - E_1}$$

2. $$\frac{dN_3}{dt} = -\gamma_{32} N_3 + R_p(N_1 - N_3)$$

3. $$\frac{dN_2}{dt} = +\gamma_{32} N_3 - \gamma_{21} N_2$$

4. $$\frac{dN_1}{dt} = -\frac{dN_2}{dt} - \frac{dN_3}{dt}, \Leftarrow N = N_1 + N_2 + N_3 \text{ is constant, so } \frac{dN}{dt} = 0.$$

$$\frac{dN_1}{dt} = \overset{+\gamma_{21} N_2}{\cancel{+\gamma_{32} N_3}} - R_p(N_1 - N_3)$$

5. At steady state, $\frac{dN_i}{dt} = 0 \ \forall i = \{1, 2, 3\}$

$$\frac{dN_3}{dt} = 0 = -\gamma_{32} N_3 + R_p(N_1 - N_3) \Rightarrow R_p N_1 = (R_p + \gamma_{32}) N_3$$

$$\frac{N_1}{N_3} = \frac{R_p + \gamma_{32}}{R_p} = 1 + \frac{\gamma_{32}}{R_p}$$

$$\frac{dN_2}{dt} = 0 = \gamma_{32} N_3 - \gamma_{21} N_2 \Rightarrow \frac{N_2}{N_3} = \frac{\gamma_{32}}{\gamma_{21}}$$

6. $$\frac{N_2 - N_1}{N_3} = \frac{N_2}{N_3} - \frac{N_1}{N_3} = \frac{\gamma_{32}}{\gamma_{21}} - \left(1 + \frac{\gamma_{32}}{R_p}\right)$$

7. Population Inversion occurs when the above equation > 0, because $N_3 > 0$ must be true.

$$\frac{N_2 - N_1}{N_3} > 0 \Rightarrow \frac{\gamma_{32}}{\gamma_{21}} - \left(1 + \frac{\gamma_{32}}{R_p}\right) > 0$$

. Divide by $\frac{1}{\gamma_{32}}$:

$$\frac{1}{\gamma_{21}} - \frac{1}{\gamma_{32}} - \frac{1}{R_p} > 0$$

This result tells us we need the decay rate γ_{21} to be low, or decay time τ_{21} to be higher, to achieve population inversion.

$$\frac{1}{\gamma_{21}} > \frac{1}{\gamma_{32}} + \frac{1}{R_p},$$

or, in terms of decay times $\tau = \frac{1}{\gamma}$ ...

$$\Rightarrow \tau_{21} > \tau_{32} + \tau_p$$