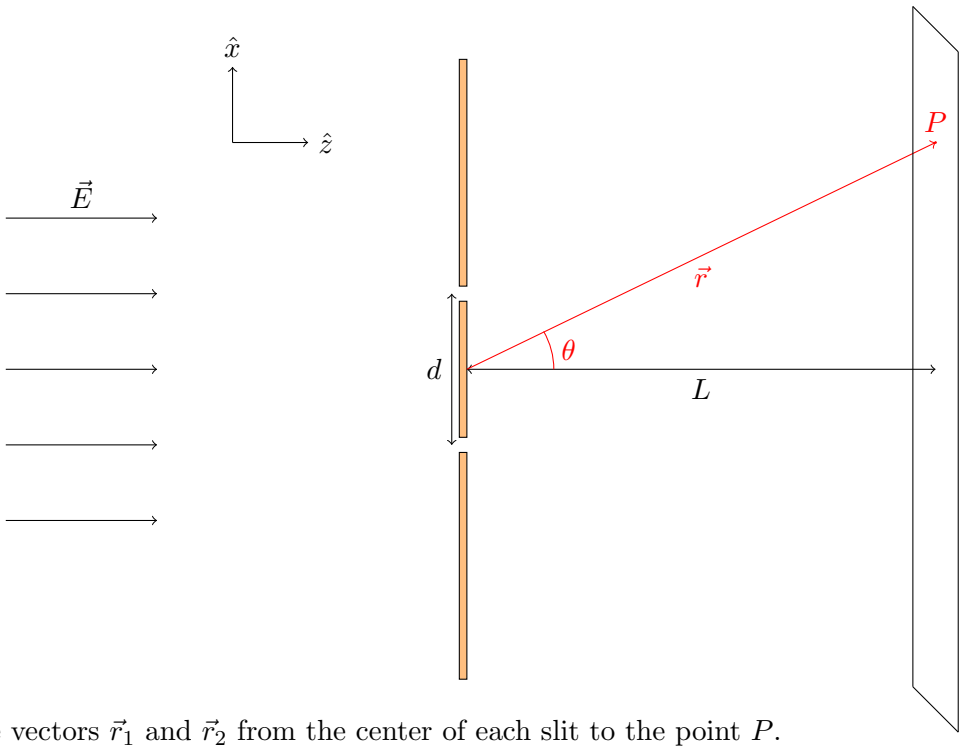
Quiz 2: Waves and Interference
January 28, 2025
Lasers and Optomechanics

Name: _____

Plane Wave incident on a Double Slit

Suppose we have two narrow slits cut a distance d apart into a opaque surface in the xy plane. Incident on the surface is a plane wave electric field $\vec{E} = E_0 \exp(ikz)\hat{x}$ propogating in the $+\hat{z}$ direction. Finally, we have another surface in the xy plane a distance L away where we would like to know the intensity $I(\theta)$, where θ is the angle from the middle of the slits to a point on the surface at L. We will assume $d \ll L$.



1. Draw the vectors $\vec{r}_1$ and $\vec{r}_2$ from the center of each slit to the point P.
2. Write expressions for $\vec{r}_1$ and $\vec{r}_2$, and $|r_1|$ and $|r_2|$ in terms of L, d and θ.
   Hint 1: Find an expression for $\vec{r}$ first.
   Hint 2: It may be helpful that $1 + \tan^2\theta = \dfrac{1}{\cos^2\theta}$
3. Write an first-order approximation for $|r_2|$ - $|r_1|$.
   Hint: Assume $d \ll L$, and use the binomial approximation $(1+\epsilon)^n \approx 1 + n\epsilon$ for $\epsilon \ll 1$.
4. Assuming that the output from each slit is a spherical wave $E_i = \dfrac{A}{|\vec{r}_i|} e^{i\vec{k}\cdot\vec{r}_i}$, write a general expression for the intensity I in terms of r_1 and r_2.
5. Simplify your expression for the intensity I in terms of θ, removing all references to r_1, r_2.
   Hint: For simplicity, expand the front fraction $\dfrac{A}{r_i}$ to zeroth order, so that $\dfrac{A}{|\vec{r}_1|} = \dfrac{A}{|\vec{r}_2|} = \dfrac{A}{|\vec{r}|}$.
6. At what θ would $I(\theta)$ first equal zero, if at all?
7. Suppose now the incident plane wave is at an angle ϕ, such that the waves struck slit 1 first. How would your new intensity expression $I(\theta, \phi)$ change compared to $I(\theta)$?