In-class Tutorial: Coupled Cavities (Part 1)

March 23, 2026

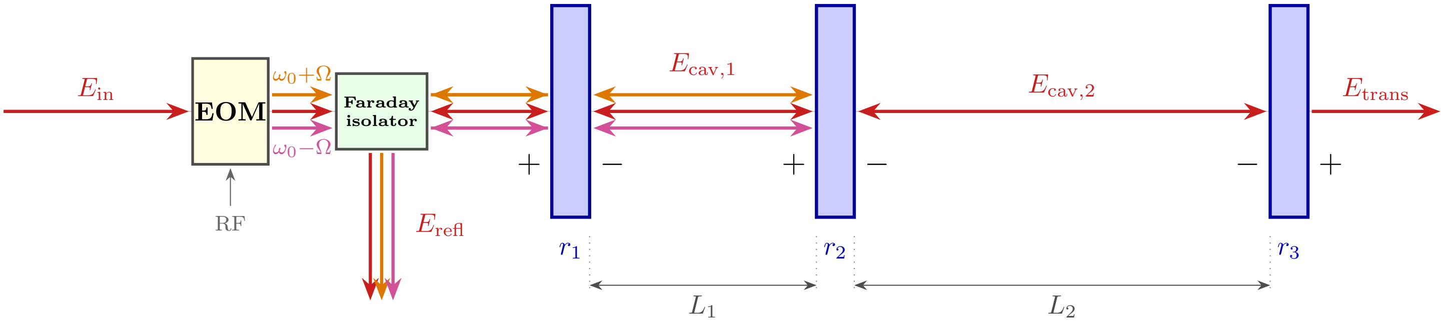
Lasers and Optomechanics

Name: ____

Coupled Cavity

A *coupled cavity* is a useful interferometer configuration for vastly enhancing the input power of a laser. Advanced LIGO employs this configuration for the *common-arm cavity* (CARM), by using a *power-recycling mirror* (PRM) in front of the Fabry-Perot arms to recycle the light reflected by the Michelson back into the arms.

The difficulty in a coupled cavity comes in the two degrees of freedom it possesses, L_1 and L_2. Sensing both L_1 and L_2 at the same time can be nontrivial. This is why the RF sidebands are typically set to co-resonate inside the first cavity, but not the second. In this tutorial, we will explore how a coupled cavity works.



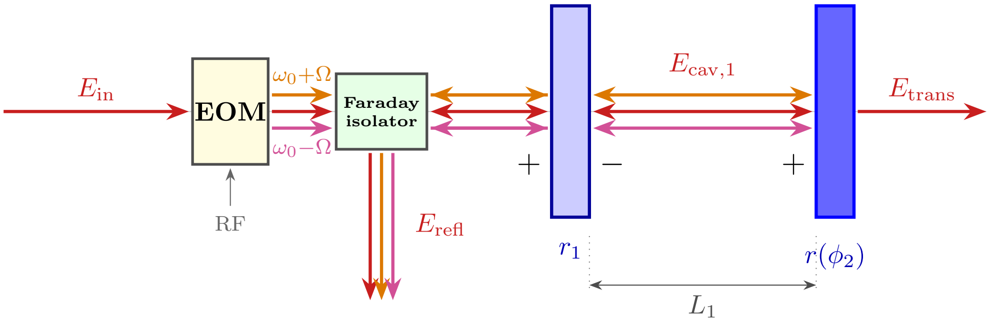


Figure 1A: Coupled-cavity interferometer, with three mirrors aligned and carrier plus one set of RF sidebands incident.

Figure 1B: Compound interferometer representing the coupled-cavity interferometer, where we have “collapsed” the r_2 and r_3 mirrors into a single mirror with reflectivity $r(\phi_2)$.

Recall the Fabry-Perot cavity reflection equation is

$$r(\phi) = \frac{E_{\rm refl}}{E_{\rm in}}(\phi) = \frac{r_i - r_e e^{-i2\phi}}{1 - r_i r_e e^{-i2\phi}} \tag{1}$$

$$g(\phi) = \frac{E_{\rm cav}}{E_{\rm in}}(\phi) = \frac{t_i}{1 - r_i r_e e^{-i2\phi}} \tag{2}$$

$$t(\phi) = \frac{E_{\rm trans}}{E_{\rm in}}(\phi) = \frac{t_i t_e e^{-i\phi}}{1 - r_i r_e e^{-i2\phi}} \tag{3}$$

where r_i, t_i and r_e, t_e are the input and end mirror reflectivities and transmissions.

Compound Interferometers and Complex Reflectivities

Above in Figure 1A, we have a diagram of a coupled cavity, featuring three aligned mirrors r_1, r_2, r_3. Between r_1 and r_2 we have a length L_1, or a single-pass phase $\phi_1 = kL_1$. Similarly, between r_2 and r_3 we accrue $\phi_2 = kL_2$. Essentially, a coupled-cavity is just taking a Fabry-Perot interferometer and placing an additional mirror in front. In fact, we can simplify our thinking by using our understanding of the Fabry-Perot reflectivity $r(\phi)$.

Figure 1B shows a *compound Fabry-Perot interferometer* representing our original coupled cavity. The compound interferometer recognizes the Fabry-Perot interferometer inside the coupled-cavity, and "collapses" the two mirrors and fields inside into a single "compound" mirror, which we say has reflectivity $r(\phi_2)$ and transmission $t(\phi_2)$. Equations 1 and 3 shows our Fabry-Perot reflectivity and transmission, which describe the field reflected and transmitted by the compound mirror.

Below we will analyze the coupled cavity, calculate the *arm gain* and *power-recycling gain*, as well as calculate the *coupled-cavity pole*. In Part 2, we'll build a *sensing matrix* for our two degrees of freedom ϕ_1 and ϕ_2. I recommend using Mathematica, sympy, or Desmos to make some of these plots at the end.

Analysis of the Coupled Cavity

We will focus only on the carrier $E_{\text{in}} = E_0 e^{i\omega_0 t}$ analysis first. The coupled-cavity typically resonates the carrier in both the first and second cavities.

1. Write the second cavity reflectivity, gain, and transmission expressions

$$r(\phi_2), g(\phi_2), t(\phi_2),$$

using Equations 1, 2, and 3, in terms of r_2, r_3, t_2, t_3, and ϕ_2.

2. Write the compound cavity reflectivity, gain, and transmission expressions

$$r(\phi_1, \phi_2), g_1(\phi_1, \phi_2), t(\phi_1, \phi_2),$$

in terms of r_1, t_1, ϕ_1, and your expressions from (1).

Here g_1 is used to emphasize this is the amplitude gain for the first cavity.

3. Expand out your terms for $r(\phi_1, \phi_2), g_1(\phi_1, \phi_2), t(\phi_1, \phi_2)$, to get the full expressions in terms of $r_1, r_2, r_3, t_1, t_2, t_3, \phi_1, \phi_2$.

4. How might you derive the amplitude gain for the second cavity $g_2(\phi_1, \phi_2)$?.

5. What should the phases for ϕ_1 and ϕ_2 be to maximize carrier power in the arm cavity?

In Advanced LIGO, the *power-recycling gain* refers to the power gain experienced inside the first cavity $|g_1(\phi_1, \phi_2)|^2$, while the *arm gain* is just the usual FP gain $g(\phi_2)$. It can be useful to consider the arm cavity gain and reflectivity ($g(\phi_2)$ and $r(\phi_2)$) here. You may need the fact that the end mirror transmission $T_3 = 5$ ppm, while the middle mirror transmission $T_2 = 1.5$ %.

6. What should the phases for ϕ_1 and ϕ_2 for the sidebands in the coupled cavity? In Advanced LIGO, the RF sideband frequency Ω is selected such that the sidebands are *anti-resonant* in the arms but resonant in the power-recycling cavity.

7. Find the coupled cavity pole ω_{cc} for carrier.

The *coupled-cavity pole* ω_{cc} is used to describe the frequency response of the coupled cavity. For simplicity, you will need to set the denominator of your coupled cavity field solutions to zero, and let $\phi_1 = 0$ and $\phi_2 = \omega_{cc} L_2 / c$. In Advanced LIGO, the arm cavity's higher finesse and longer length let it's dynamics dominate over the first cavity, which is why $\phi_1 = 0$ is a decent approximation.