In-class Tutorial: Pound-Drever-Hall Fabry-Pérot
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Lasers and Optomechanics

Name: ____

Disadvantages of Pound-Drever-Hall Locking

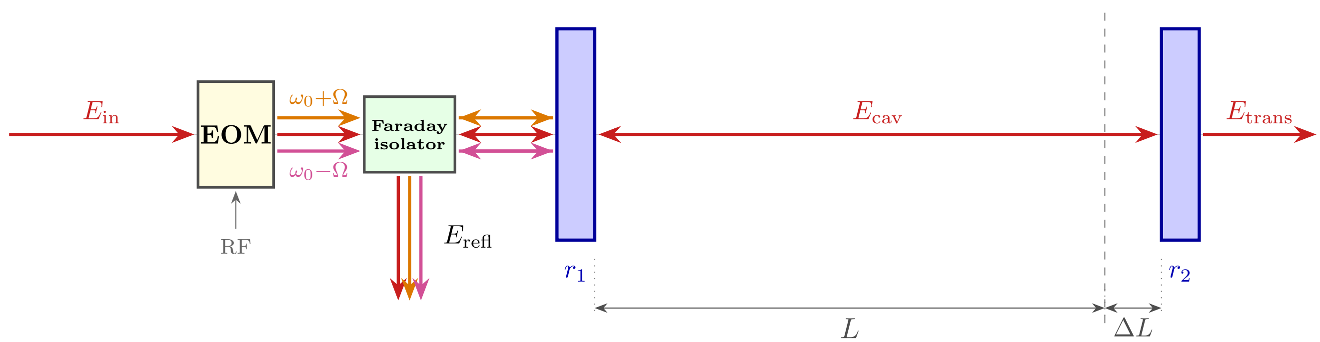
Previously we investigated *dither locking* as a method of sensing the resonance of a Fabry-Pérot cavity. Dither locking is a powerful technique for sensing the resonance of a cavity, far better than *half-fringe locking* where the cavity was held at half-power in transmission.

Still, dither locking has some disadvantages. First, it required a constant length modulation of the cavity, which can cause some nonlinearities, uses up actuation range, limits us to actuating at low frequencies (< 100 kHz), and allows noise couplings through our system.

Additionally, if we carefully analyze our error signal, the zero point may not be exactly at resonance. A good cavity resonance error signal should tell us two things:

1. How far away we are from true resonance ($\phi = 0$),
2. Which direction we need to move to get to resonance ($\phi > 0$ or $\phi < 0$).

If we have a overcoupled cavity, as we often do in practice, the dither lock may take us to just slightly off resonance.



Above we have a diagram of a Fabry-Pérot cavity set up for Pound-Drever-Hall locking. Recall the FP cavity reflection equation is

$$r(\phi) = \frac{E_{\text{refl}}}{E_{\text{in}}}(\phi) = \frac{r_1 - r_2 e^{-2i\phi}}{1 - r_1 r_2 e^{-2i\phi}} \tag{1}$$

Pound-Drever-Hall Locking

Pound-Drever-Hall circumvents some of these problems by imposing *phase-modulated radio-frequency (RF) sidebands* which co-propagate with the carrier. These sidebands, labeled $\omega_0 + \Omega$ and $\omega_0 - \Omega$ above, are typically tuned such that if the carrier frequency ω_0 resonates in the cavity, the RF sidebands are *antiresonant* in the cavity.

We will explore the setup of the PDH lock in this note. I recommend using Mathematica, sympy, or Desmos to make some of these plots at the end.

Derivation of the PDH error signal

Suppose we start with our carrier light $E_{\text{in}} = E_0 e^{i\omega_0 t}$. Then, we send this light through the electro-optic modulator (EOM) to create the phase-modulated sidebands with modulation depth Γ and modulation frequency Ω.

1. What is the total electric field incident on the front of the cavity?

(It can be helpful to explicitly label your carrier, upper sideband, and lower sideband in the total field).

Next, the light incident on the cavity enters and is reflected by the cavity. Each of our three fields will interact with the cavity differently.

2. Write down three single-pass phases $\phi_0, \phi_{\text{usb}}, \phi_{\text{lsb}}$ that the carrier, upper sideband, and lower sideband, will experience as they traverse the cavity, in terms of the cavity length L, and carrier and RF frequencies ω_0 and Ω.

Now that we have our phases, let's use them to describe the total light that is reflected by the cavity.

3. Write down the cavity total reflected field E_{refl} in terms of the reflection function $r(\phi)$ and $\phi_0, \phi_{\text{usb}}, \phi_{\text{lsb}}$.

Now we have our total reflected field, let's make our power measurement on our photodetector.

4. Write down the cavity total reflected power $P_{\text{refl}}(t)$ in terms of the reflection function $r(\phi)$ and $\phi_0, \phi_{\text{usb}}, \phi_{\text{lsb}}$.

You should end up with nine terms, since you are combining three fields. At the end, you can set $\Gamma^2 = 0$, to assume small modulation depth.

We have our long $P_{\text{refl}}(t)$ term. Look at the terms which oscillate at 1Ω. Group the $e^{i\Omega t}$ and $e^{-i\Omega t}$ terms. Set the coefficient for $e^{-i\Omega t}$ equal to some complex number z, and the coefficient for $e^{i\Omega t}$ equal to y.

5. What is z and y? Is there any connection between z and y? What is it?

6. Rewrite P_{refl} in terms of z and y.

Let's try our demodulation in a new way. We usually find P^I and P^Q by integrating $P(t)$ over one cycle of $\cos(\Omega t)$ and $\sin(\Omega t)$, because this gives us a single real number for our two quadratures and is how these measurements are usually made. But we can combine these equations into a single demodulation over $e^{i\Omega t}$ to recover our 1Ω demodulated response function $P^{\Omega}(\phi, \phi_{\text{usb}}, \phi_{\text{lsb}})$.

7. Show we can rewrite the demodulation in this way.

8. Calculate the 1Ω demodulated power $P^{\Omega}_{\text{refl}}(\phi_0, \phi_{\text{usb}}, \phi_{\text{lsb}})$. What can we see pops out?

P^{Ω}_{refl} is our PDH error signal. Assume the RF sidebands are antiresonant, so $\phi_{\text{usb}} = \phi_0 + \frac{\pi}{4}, \phi_{\text{lsb}} = \phi_0 - \frac{\pi}{4}$.

9. Calculate the derivative of P^{Ω}_{refl} with respect to ϕ_0, then evaluate it about zero.

This is called the *discriminant* or *optical gain* of the error signal, in units of watts per radian.

10. Plot the real and imaginary parts of $P^{\Omega}_{\text{refl}}(\phi_0, \phi_{\text{usb}}, \phi_{\text{lsb}})$ as a function of $\phi_0 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

Plot the discriminant $\frac{P^{\Omega}_{\text{refl}}}{d\phi_0}$ about $\phi_0 = 0$ for comparison.

Choose a moderate finesse cavity with length $L = 1$ m, $T_1 = T_2 = 20\%$, and $\Gamma = 0.1$ radians.
State what the modulation frequency Ω must be to be exactly antiresonant in the $L = 1$ m cavity.

11. Plot $P^{\Omega}_{\text{refl}}(\phi_0, \phi_{\text{usb}}, \phi_{\text{lsb}})$ as a phasor as a function of $\phi_0 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

What shape do you get?