Quiz 1: Complex Numbers Review
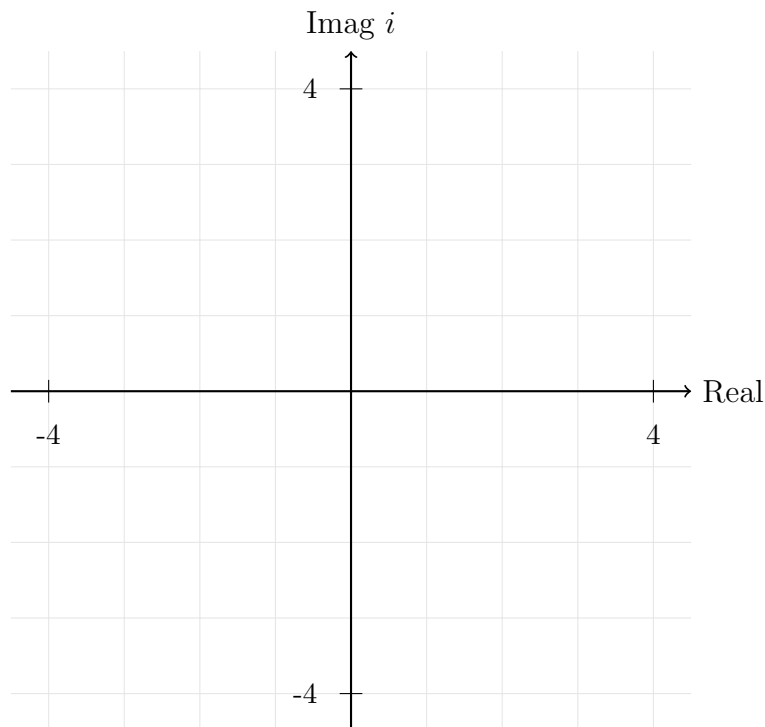January 14, 2025
Lasers and Optomechanics

Name: _____

Question 1: Complex Geometry

1. Plot and label the point $z = -2 + 2i$.

2. Write z in terms of its polar coordinates r and θ.

3. Plot and label its complex conjugate z^*.

4. Plot and label the sum $z + z^*$

5. Plot and label the difference $z - z^*$

6. Plot and label the product $\dfrac{z^2}{2}$

7. Plot and label the product $\dfrac{|z|^2}{2}$
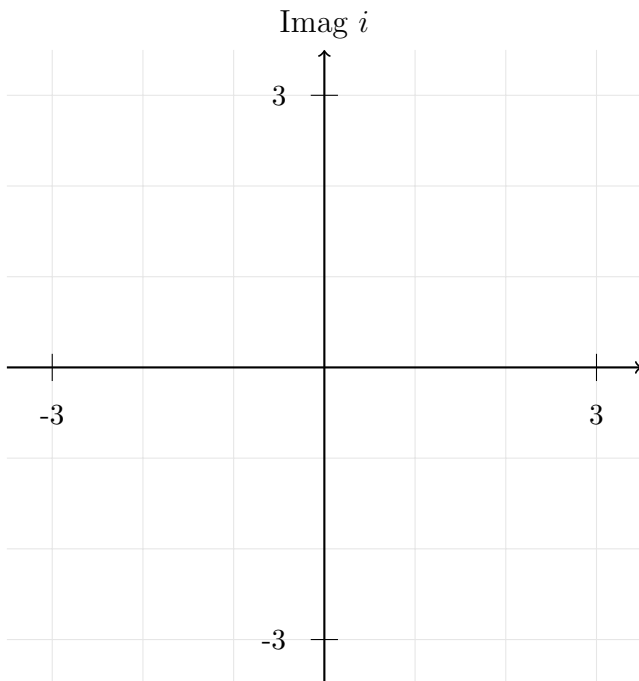
Question 2: Complex Functions

Suppose we have a complex functions z_1, z_2, and z_3 defined as a function of $\phi \in [0, 2\pi]$:

$$z_1(\phi) = 2 + e^{i\phi} \tag{1}$$

$$z_2(\phi) = \frac{1}{2}(e^{i\phi} + 3e^{-i\phi}) \tag{2}$$

$$z_3(\phi) = \frac{3\sqrt{2}ie^{i\frac{\pi}{4}}}{2}(e^{i\phi} + e^{-i\phi}) \tag{3}$$

1. Plot and label z_1, z_2, and z_3 over $\phi \in [0, 2\pi]$.

2. What shapes do z_1, z_2, and z_3 form?

3. Which way are z_1 and z_2 rotating as ϕ increases?

4. How can you know which way z_1 and z_2 are rotating? Justify it with a calculation.

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Question 3: Fourier Transform

Calculate the Fourier Transform of $x(t) = 2\cos(t) + \sin(t)$. Recall that $X(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt$