In-class Tutorial: Fabry-Pérot Dark Offset Transfer Function
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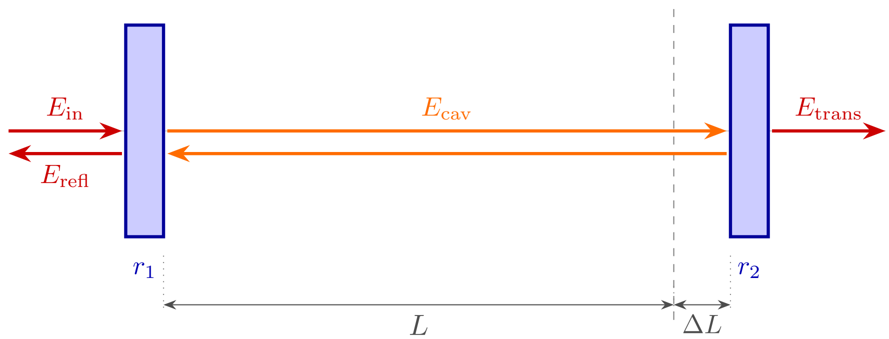
Lasers and Optomechanics

Name: ____

Sensing the Fabry-Pérot's Resonance

Suppose we have a Fabry-Pérot interferometer. We would like to know whether the Fabry-Pérot is on resonance. Here we will explore possibilities for sensing the resonance of a Fabry-Pérot cavity, especially in the presence of length disturbances $\Delta L(t)$.



Above we have a diagram of a Fabry-Pérot cavity, with a nominal length L, but also some static offset ΔL. We will explore our capability to monitor and restore our lock in the case of a static offset.

Recall that field transfer functions of the lossless Fabry-Perot were

$$\frac{E_{\text{refl}}}{E_{\text{in}}} = \frac{r_1 - r_2 e^{-i2\phi}}{1 - r_1 r_2 e^{-i2\phi}}, \qquad \frac{E_{\text{cav}}}{E_{\text{in}}} = \frac{t_1}{1 - r_1 r_2 e^{-i2\phi}}, \qquad \frac{E_{\text{trans}}}{E_{\text{in}}} = \frac{t_1 t_2 e^{-i\phi}}{1 - r_1 r_2 e^{-i2\phi}} \tag{1}$$

Part 1: Transmitted Power

The easiest way to tell if light is resonating in the cavity is to look at the transmitted beam. Intuitively, there is a maximum transmitted light when the light in the cavity is maximized.

1. Derive an expression for the transmitted *power* $P_{\text{trans}}(\phi)$.
2. What is the maximum transmitted power at resonance $P_{\text{trans}}(0)$?
3. Starting at resonance, what happens to the transmitted power when the cavity gets longer?
4. Starting at resonance, what happens to the transmitted power when the cavity gets shorter?
5. Would it be possible to use the transmitted power to *restore* resonance in case of some disturbance ΔL? How might you try to go about it?

Part 2: Reflected Power

Another option we can analyze is the power in reflection of the cavity P_{refl}:

1. Derive an expression for the reflected power $P_{\text{refl}}(\phi)$. What is the reflected power at resonance?
2. Set $r_2 = r_1 = r$. Does $P_{\text{refl}}(\phi)$ simplify at all? What value do you get at resonance?
3. Starting at resonance, what happens to the reflected power when the cavity gets longer? Shorter?
4. Can you use reflected power to restore resonance?

Part 3: Reflected Field

From looking at the transmitted and reflected power, we cannot directly use the power signals from the interferometer to hold onto resonance. But we have ignored a crucial signal coming from the interferometer, the phase. Here we will calculate the change in reflected field $\frac{dE_{\text{refl}}}{d\phi}$.

1. Derive an expression for the change in reflected field with respect to phase ϕ: $\frac{dE_{\text{refl}}}{d\phi}(\phi)$.

   Answer: $\frac{dE_{\text{refl}}}{d\phi}(\phi) = \frac{i2r_2(1-r_1^2)e^{-i2\phi}}{(1-r_1r_2e^{-i2\phi})^2}$

2. Set $r_2 = r_1 = r$. What is $E_{\text{refl}}(0)$? What is $\frac{dE_{\text{refl}}}{d\phi}(0)$?

3. What happens to the reflected field when the cavity gets longer? What about shorter?

4. Could you use the reflected field to restore resonance? What would you need in order to do so?

Part 4: Dither Locking

In Part 3, we found that it could be possible to use the reflected field phase. But how may we measure this signal using photodetectors?

One solution is to introduce a modulation of the end mirror $\Delta x(t)\cos(\omega t)$. By introducing a known length modulation of the intracavity field, we can "rotate" our phase signal into the amplitude quadrature, where it can be measured at the frequency ω.

Below, we'll go through the math of how dither locking works for the reflected field.
The process is roughly:
A) Find the carrier's contribution to the reflected field (already done in Part 3).
B) Find the carrier field incident on the modulating mirror.
C) Create the audio sidebands.
D) Find the sidebands' contribution to the reflected field.
E) Find the total power signal $P_{\text{refl}}(t)$.
F) Demodulate that power signal at ω.
What we should find is a power signal at ω that is proportional to our static offset $\phi = k\Delta L$. If $\phi = 0$, our audio sideband fields should cancel exactly, giving us zero. But if $\phi \neq 0$, we should get a power signal that changes sign at ω, which tells us which way to move our mirror to recover our resonance.

1. First, find the transfer function from the end mirror E_{cav2} to the reflected field E_{refl}.

   Answer: $\frac{E_{\text{refl}}}{E_{\text{cav2}}}(\phi) = \frac{t_1e^{-i\phi}}{1-r_1r_2e^{-i2\phi}}$

2. Find the carrier field incident on the end mirror M_2.

3. Apply the length modulation $\Delta x(t)\cos(\omega t)$ to find the total field immediately reflected off the end mirror E_{cav2}. *Hint: There should be three fields.*

4. Apply your transfer function $\frac{E_{\text{refl}}}{E_{\text{cav2}}}(\phi)$ from (1) to your total E_{cav2} from (3) to find the total reflected field. *Hint: It may be easier to consider the accrued phases for carrier ϕ and for the audio sidebands $\phi \pm \eta$, where $\phi = \omega_0 L/c$ and $\eta = \omega L/c$.*

5. Calculate the total reflected power $P_{\text{refl}}(t)$. Let $\Delta x^2 \to 0$ for small length modulations.

6. Demodulate the total reflected power at ω to calculate P^I_{refl} and P^Q_{refl}.